Improving the stability of Kalman filters with Posit arithmetic

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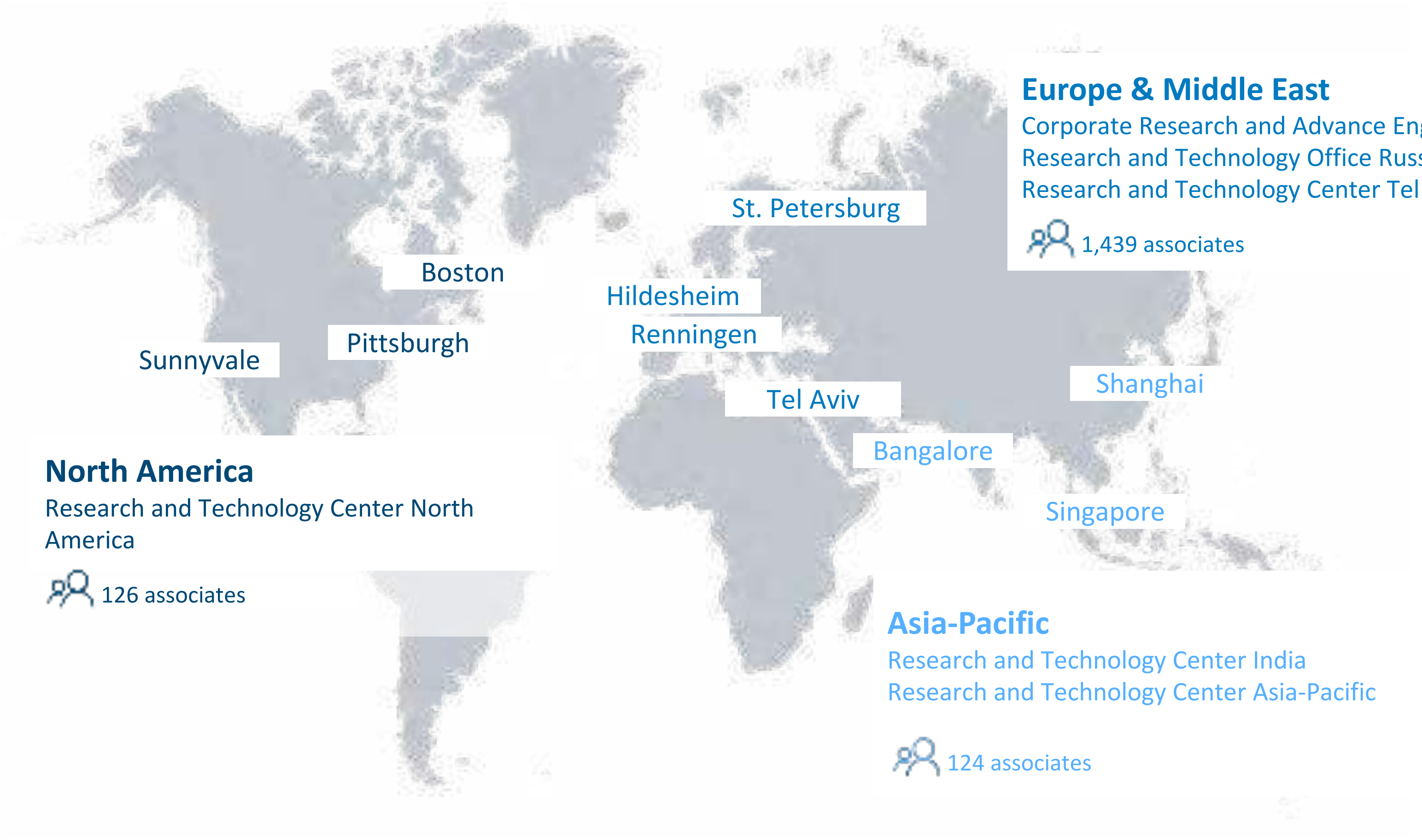

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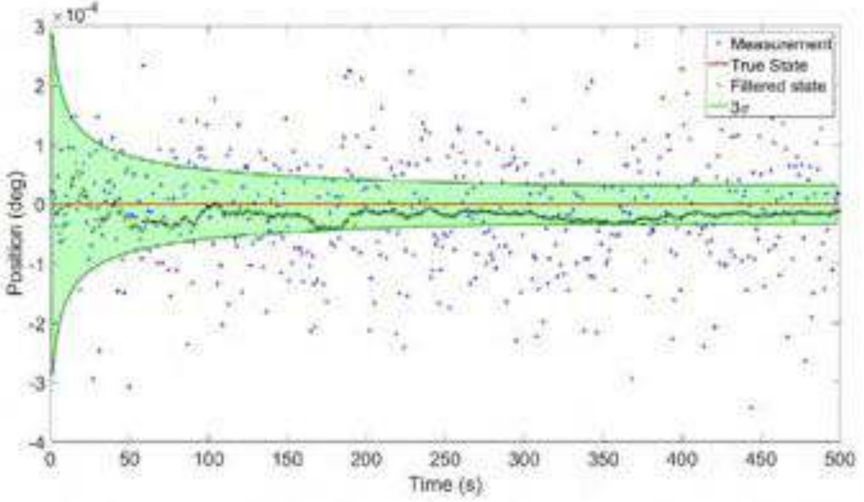
Improving the stability of Kalman filters with Posit arithmetic
Kalman Filters: Introduction

- Kalman filter is an algorithm that provides accurate state estimation of a dynamical system using measurements that are noisy
- It combines the noisy measurement data with an analytical model of the system

- Advantages:
  - Highly accurate
  - Real-time
  - Robust
  - Adaptable
  - Low computational cost



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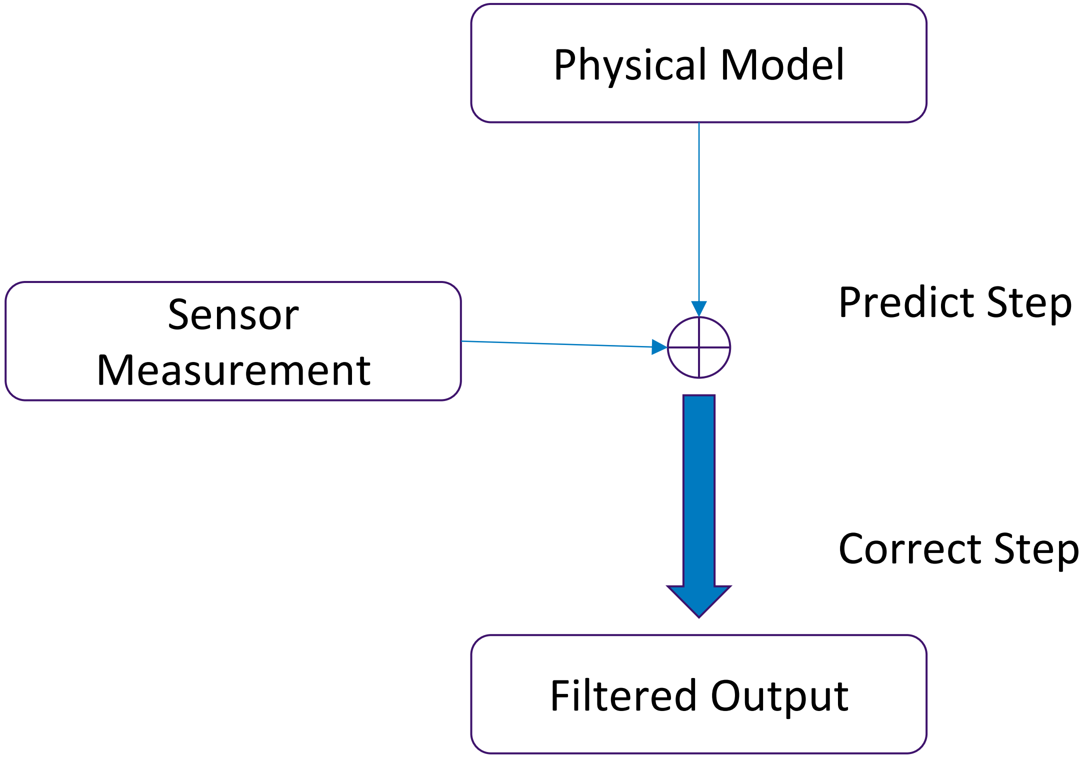
Kalman Filters: Algorithm

- Classic linear Kalman filter:



$$\hat{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\hat{\mathbf{P}} = \mathbf{F}\mathbf{P}\mathbf{F}^T + \mathbf{Q}$$

$$\mathbf{y} = \mathbf{z} - \mathbf{H}\hat{\mathbf{x}}$$

$$\mathbf{K} = \hat{\mathbf{P}}\mathbf{H}^T(\mathbf{H}\hat{\mathbf{P}}\mathbf{H}^T + \mathbf{R})^{-1}$$

$$\mathbf{x} = \hat{\mathbf{x}} + \mathbf{K}\mathbf{y}$$

$$\mathbf{P} = (\mathbf{I} - \mathbf{K}\mathbf{H})\hat{\mathbf{P}}$$

$\mathbf{x}$ is the state estimate
$\mathbf{P}$ is the state covariance matrix
$\mathbf{F}$ is the state transition function
$\mathbf{Q}$ is the process covariance matrix
$\mathbf{H}$ is the measurement function
$\mathbf{z}$ is the measurement
$\mathbf{y}$ is the measurement residual
$\mathbf{R}$ is the measurement covariance matrix
$\mathbf{K}$ is the Kalman filter gain
$\mathbf{I}$ is the identity matrix

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Kalman Filters: Issues

- Kalman Filters suffer from stability issues due to approximate real number arithmetic using floating points
- Research over the years has indicated the following common modes of failures:
  - High value of initial state covariance matrix (P_0)
  - Highly accurate measurement (R)
  - Very low process noise (Q)
  - If a measurement (H) is correlated to more than one state.
  - Non-Symmetric update of the State covariance matrix

- *Hint: When the state covariance matrix P ceases to be positive definite, it indicates some kind of failure*

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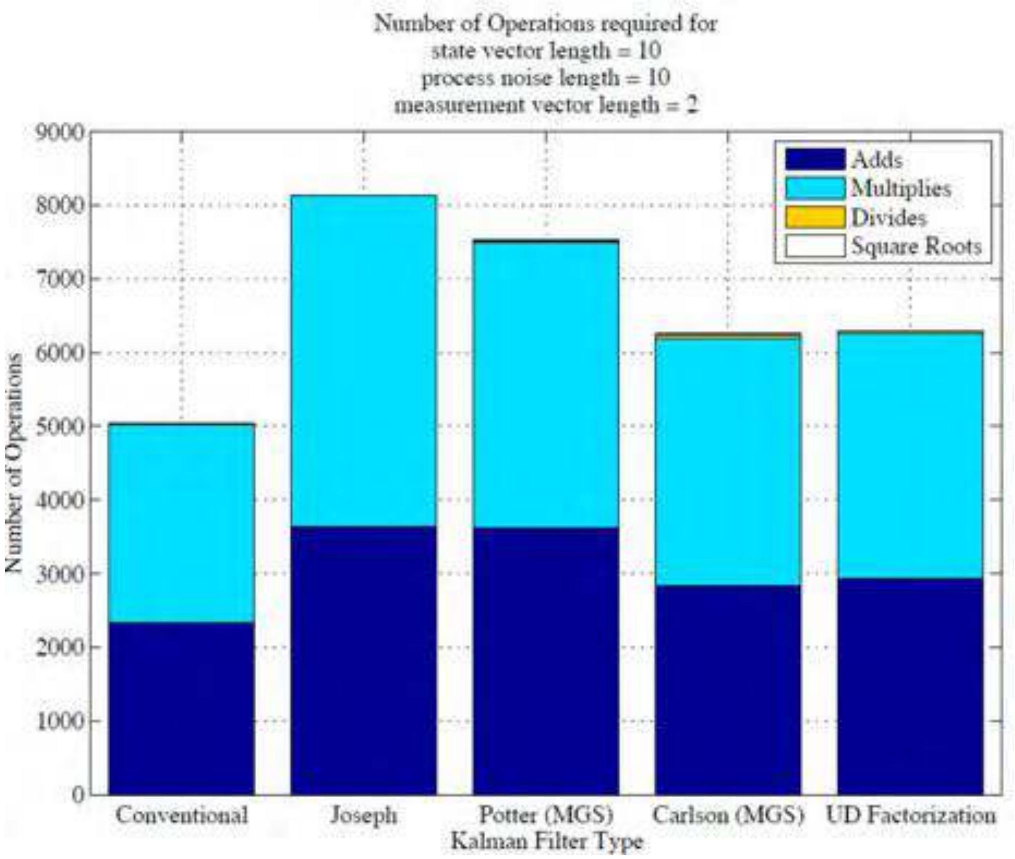
Kalman Filters: Implementation Variants

- Different mechanizations (mathematical variants) of KF have been proposed to mitigate these numerical issues



Conventional mechanization is the least compute intensive!

Source: Analysis of Square-Root Kalman Filters for Angles-Only Orbital Navigation and the Effects of Sensor Accuracy on State Observability. J. Schmidt

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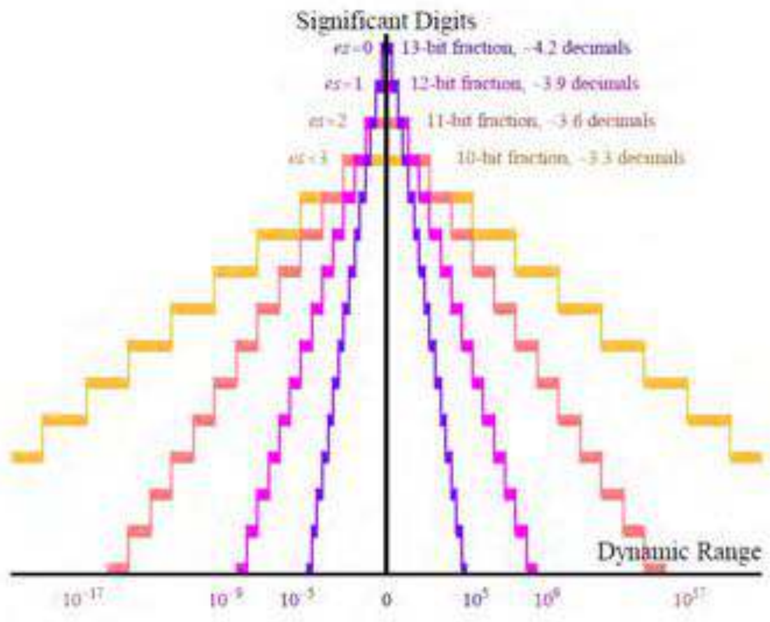
Posit Arithmetic: Introduction

- Drop-in replacement for IEEE-754 Floating point numbers
- Possible advantages include:
  - Higher dynamic range
  - Higher accuracy
  - Bitwise identical results across systems
  - Simpler hardware
  - Simpler exception handling



Source: Beating Floating Point at its Own Game: Posit Arithmetic
John L. Gustafson1, Isaac Yonemoto2

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Experiment Setup

- Taking double precision as golden reference, we plug in and test the stability of two applications with different configurations of 32-bit Posits
- We then compare the performance of Posit based filter with IEEE float based filter
- We develop a heuristic to choose the right Posit configuration

- Test case 1:
  - Estimating the position and velocity of a rotating shaft
  - The sensor is assumed to be very accurate leading to very small numerical covariance matrix

- Test case 2:
  - Estimating the relative position and velocity of 2 rotating shafts
  - The experiment is set up such that the filter covariance

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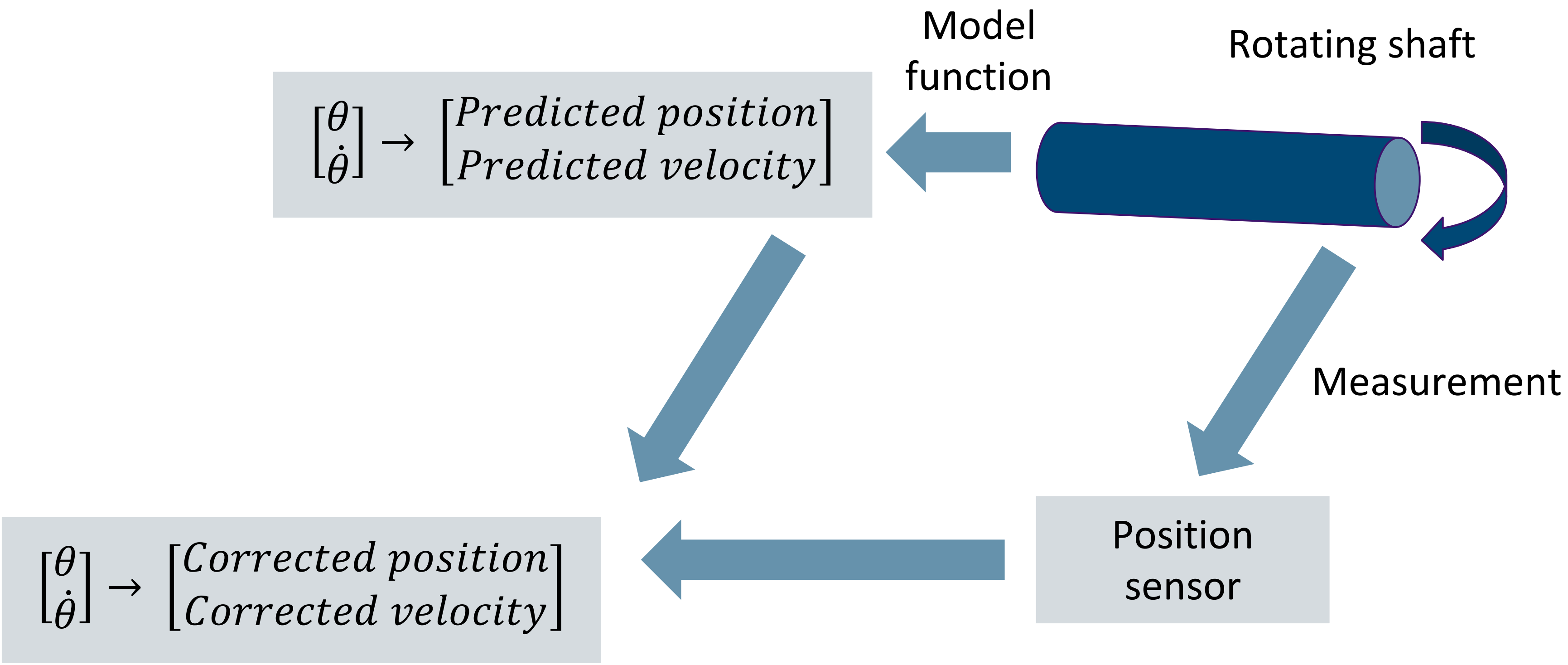
Case study I



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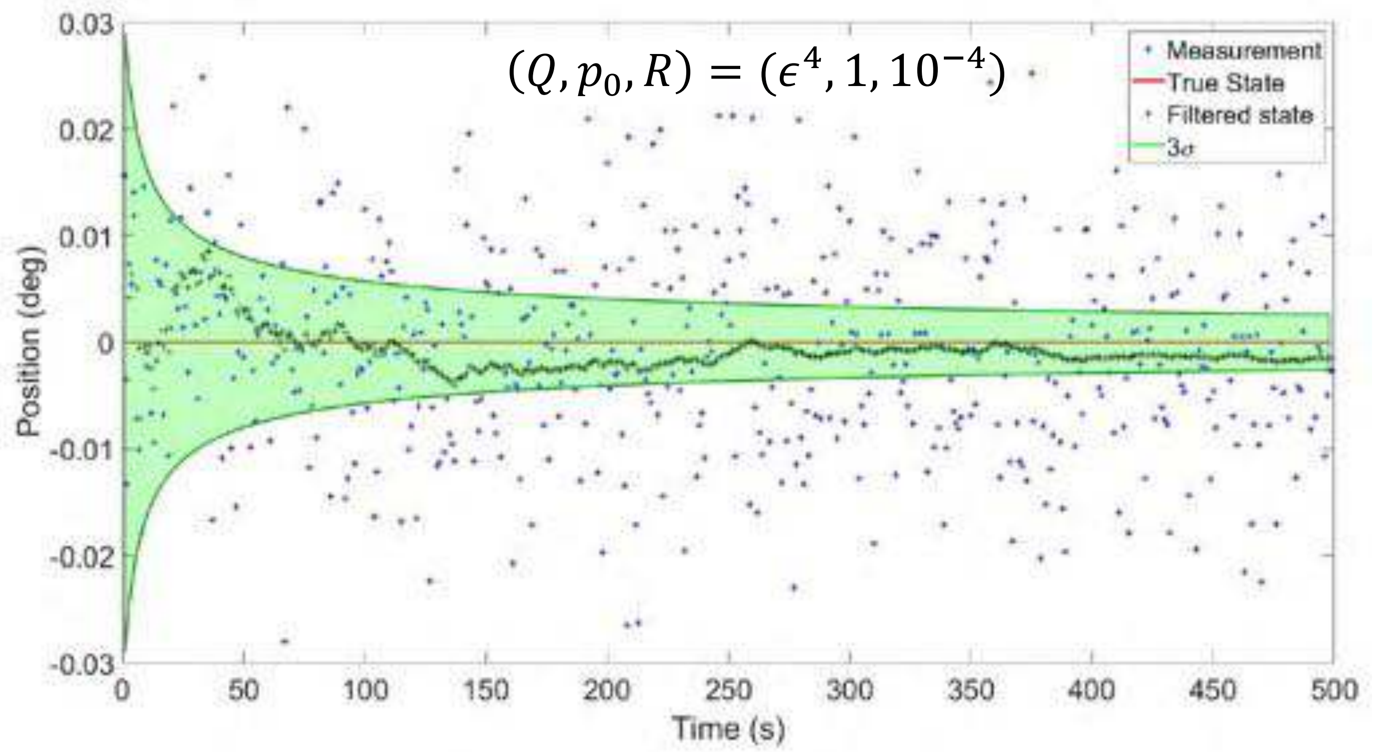
Case study I → Results



Normal working of filter for this test case

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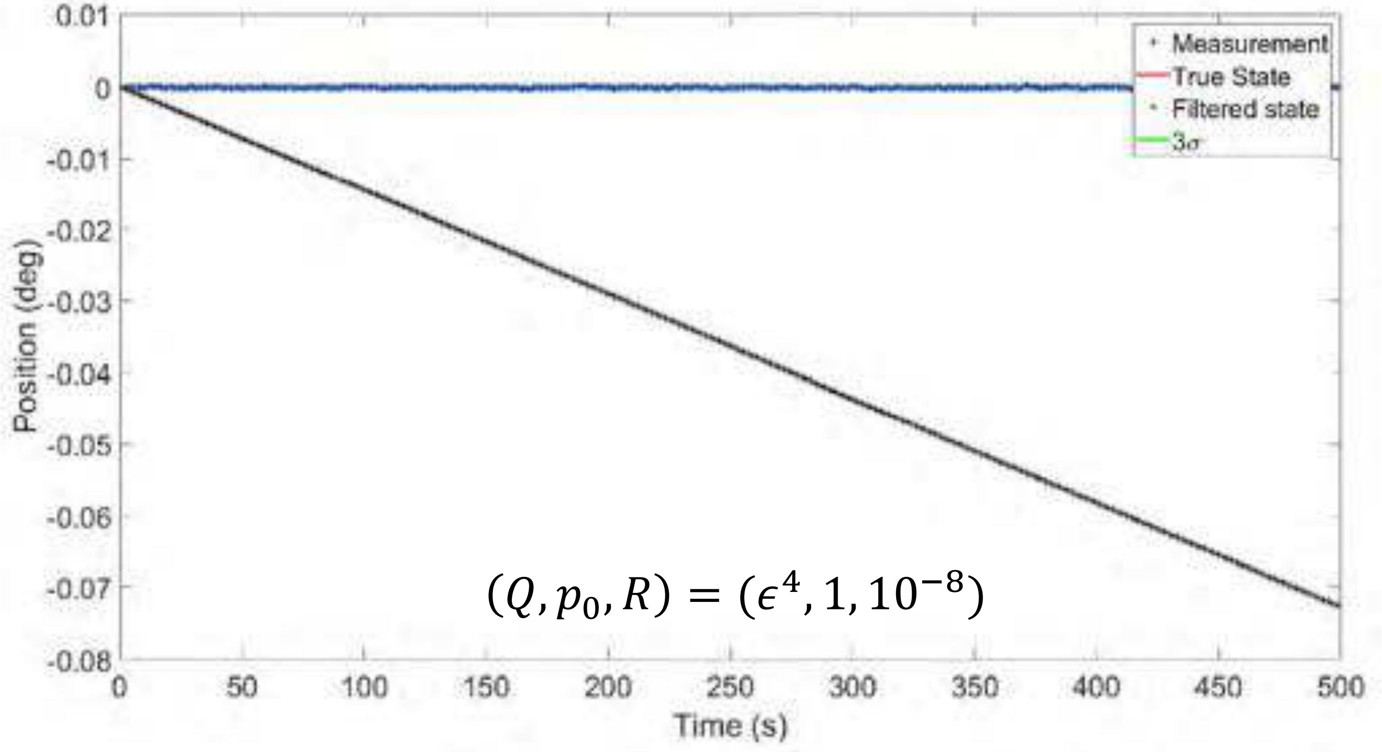
Case study I → Results



Failure of filter due to round-off errors in IEEE Floats

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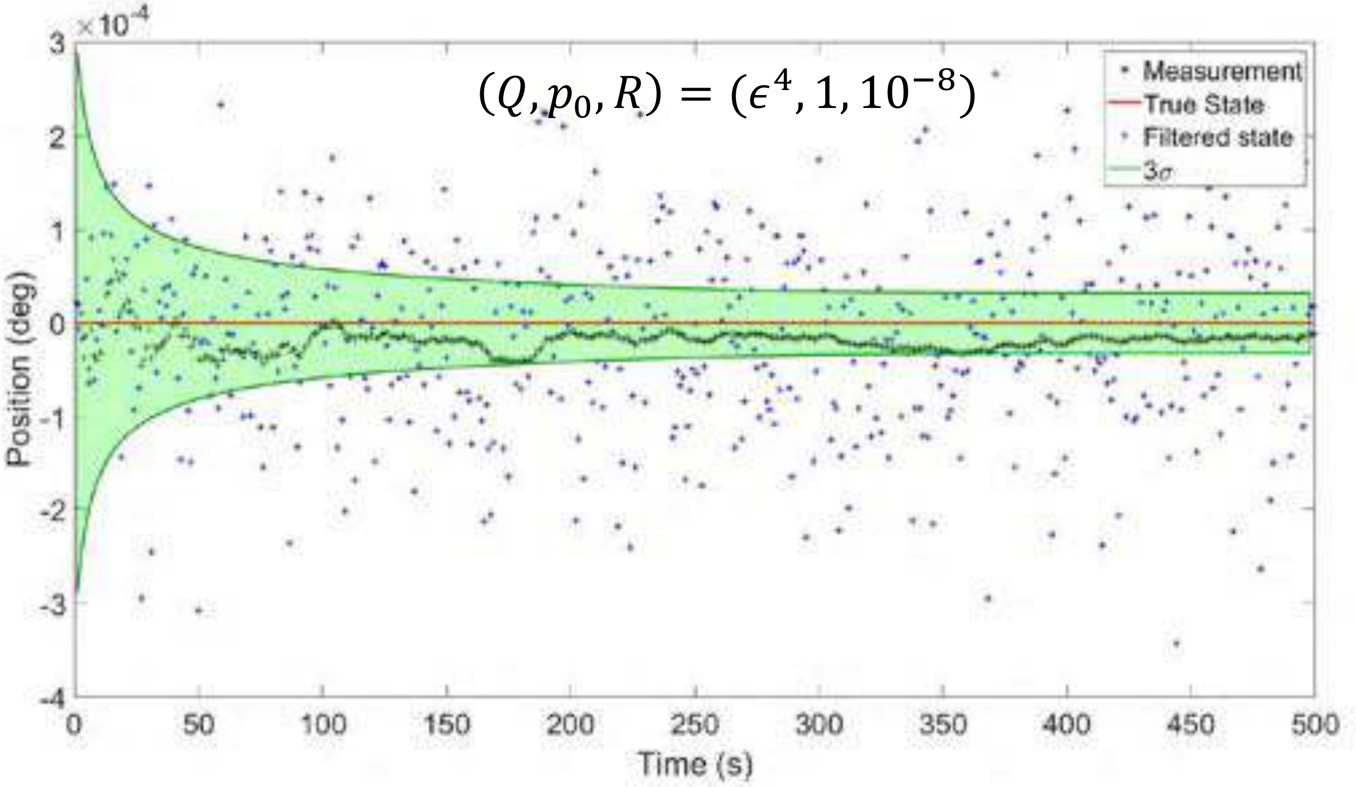
Case study I → Results



Normal working with Posit 32,1 which failed with IEEE Floats

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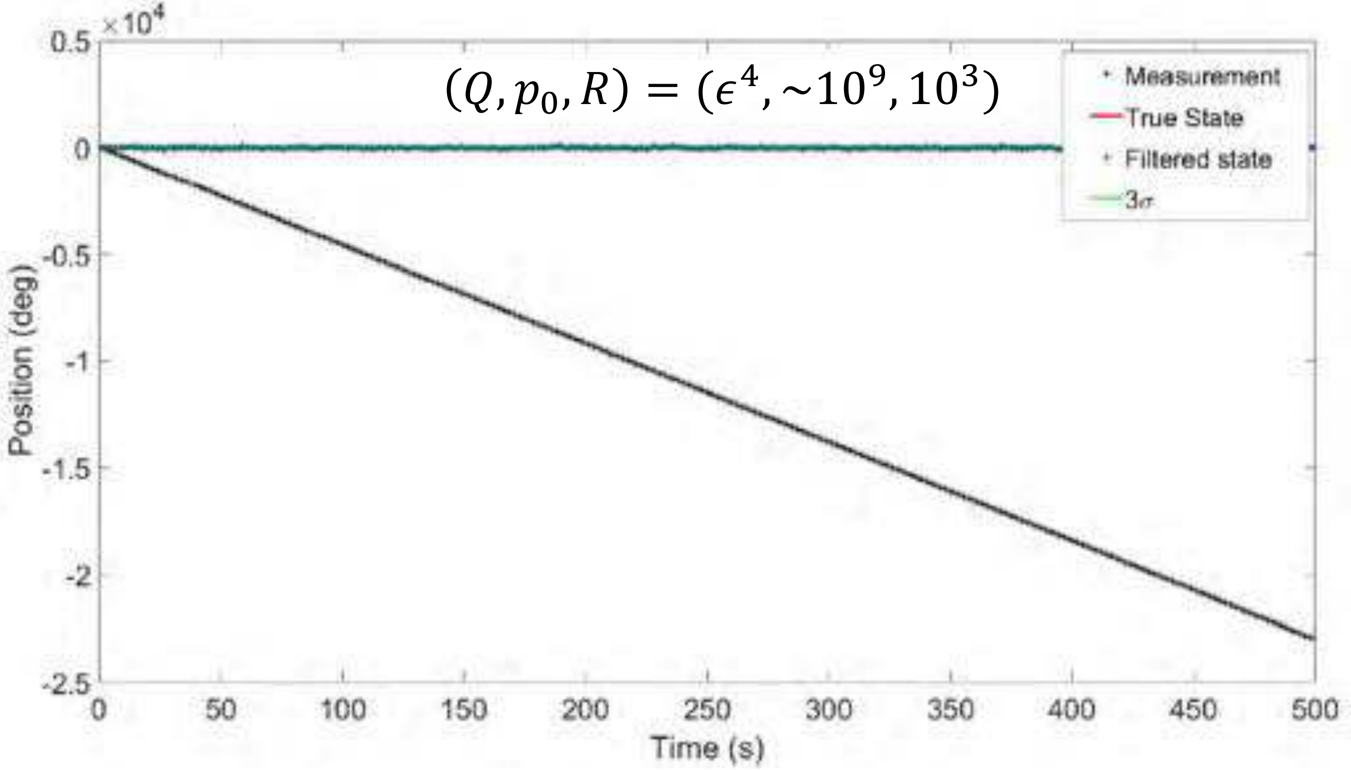
Case study I → Results



Failure of filter with Posit 32,1 due to lower precision in the instable region

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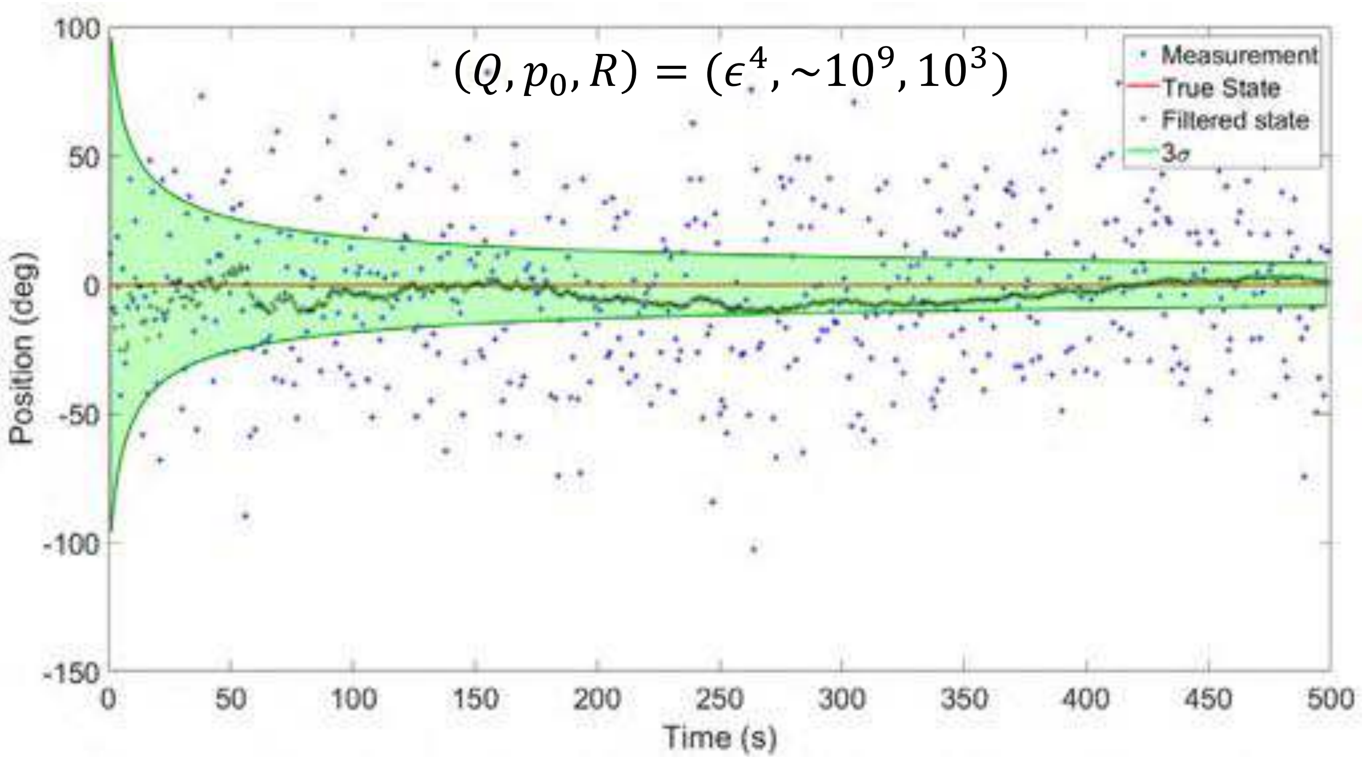
Case study I → Results



Normal working with Posit 32,4 which failed with Posit 32,1

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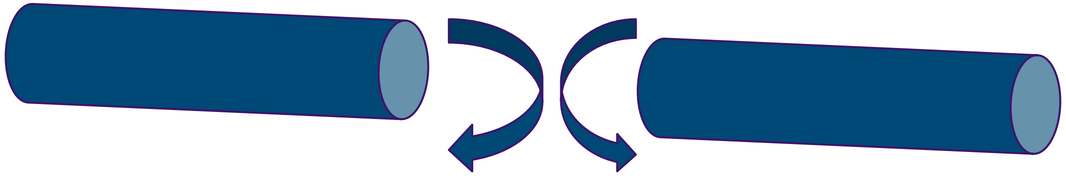
Case study II

$$\begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} \rightarrow \begin{bmatrix} \mathit{Predicted\ Angular\ position\ 1} \\ \mathit{Predicted\ Angular\ velocity\ 1} \\ \mathit{Predicted\ Angular\ position\ 2} \\ \mathit{Predicted\ Angular\ velocity\ 2} \end{bmatrix}$$

Model function



Measurement

Position sensor measures relative position

$$\begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} \rightarrow \begin{bmatrix} \mathit{Corrected\ Angular\ position\ 1} \\ \mathit{Corrected\ Angular\ velocity\ 1} \\ \mathit{Corrected\ Angular\ position\ 2} \\ \mathit{Corrected\ Angular\ velocity\ 2} \end{bmatrix}$$

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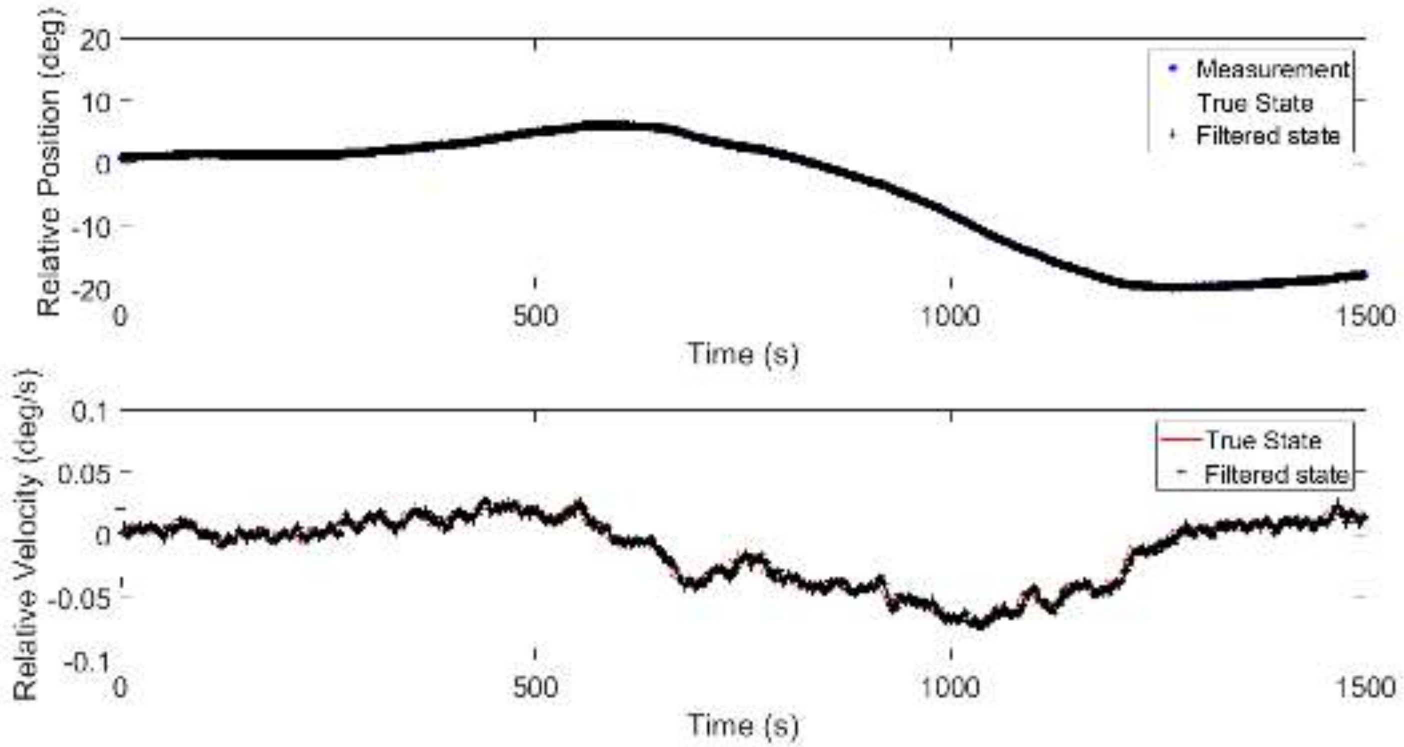
Case study II → Results



Normal working with Double precision Floats. Stability maintained till 1500 seconds

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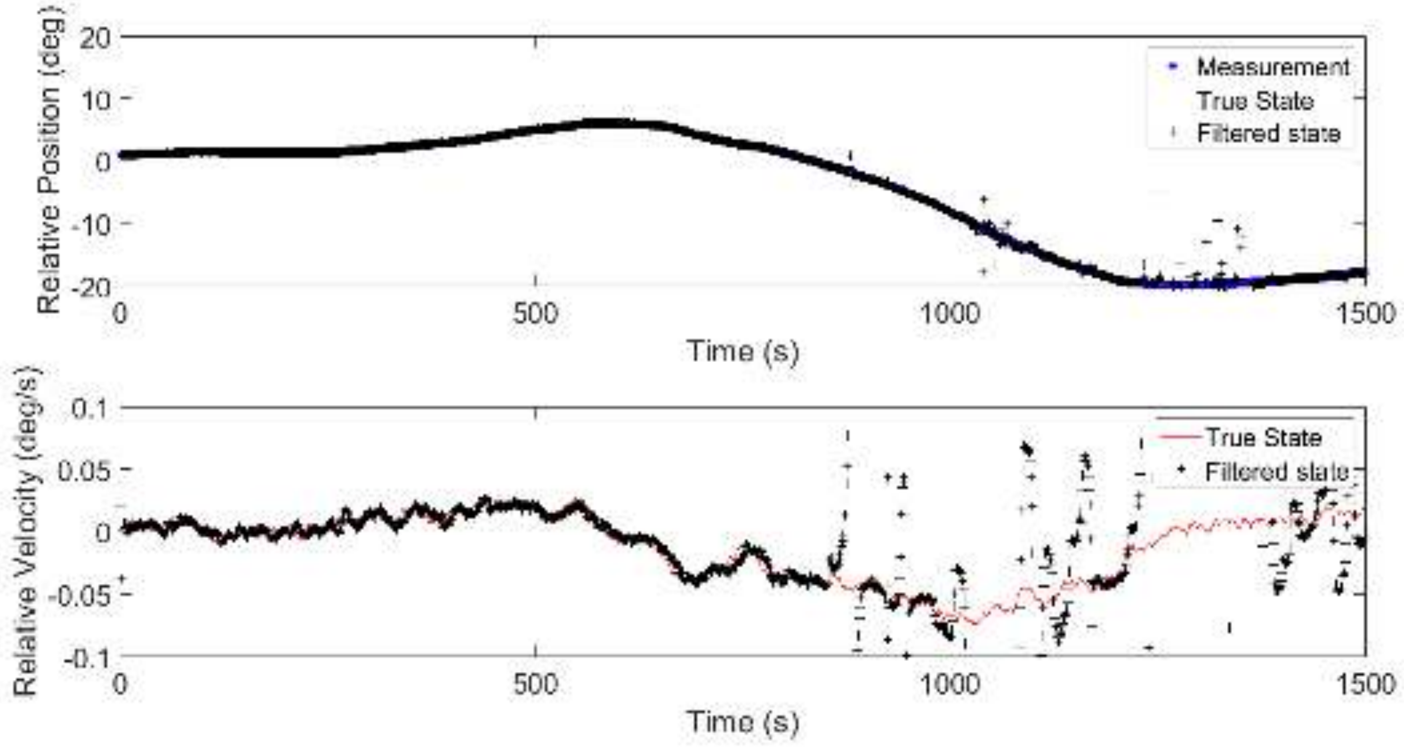
Case study II → Results



32-bit Float implementation starts diverging around 800 second mark

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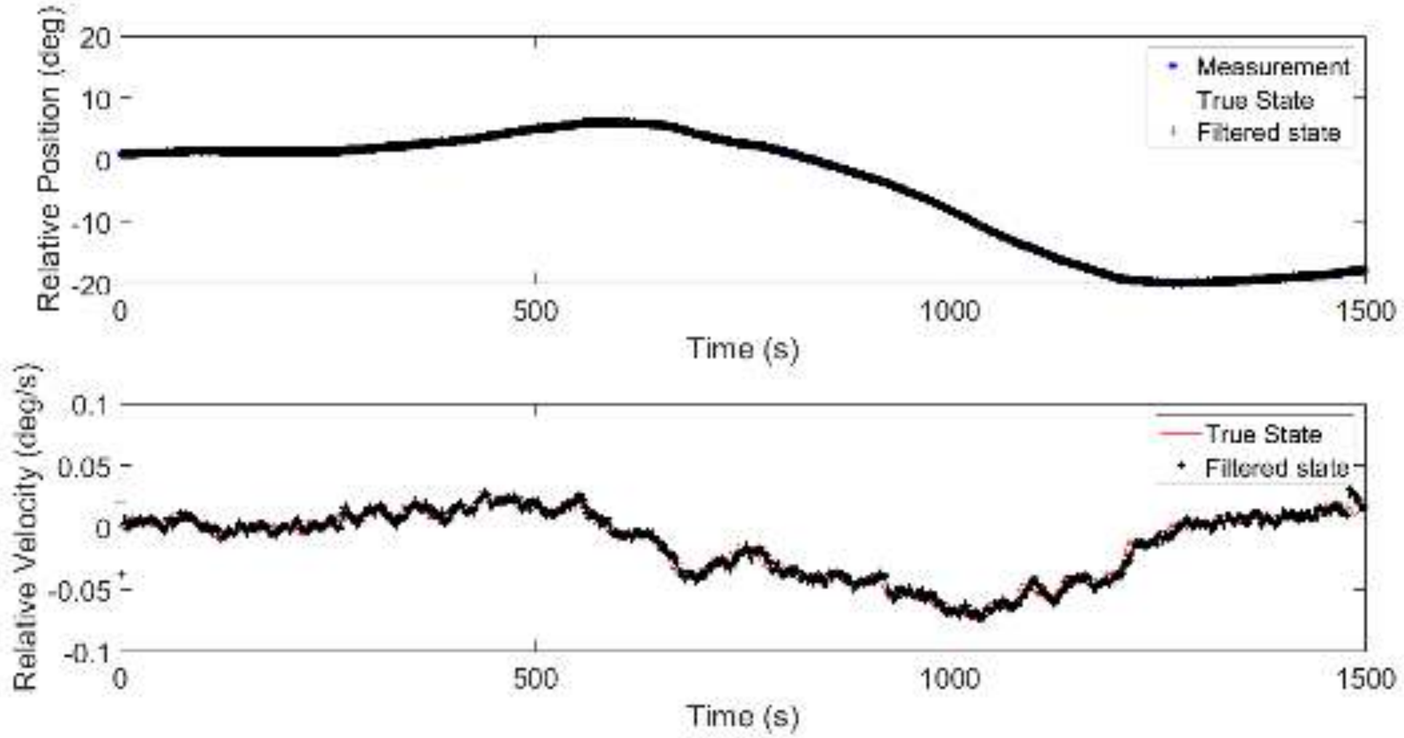
Case study II → Results



32,4 Posit implementation reproduces double precision results

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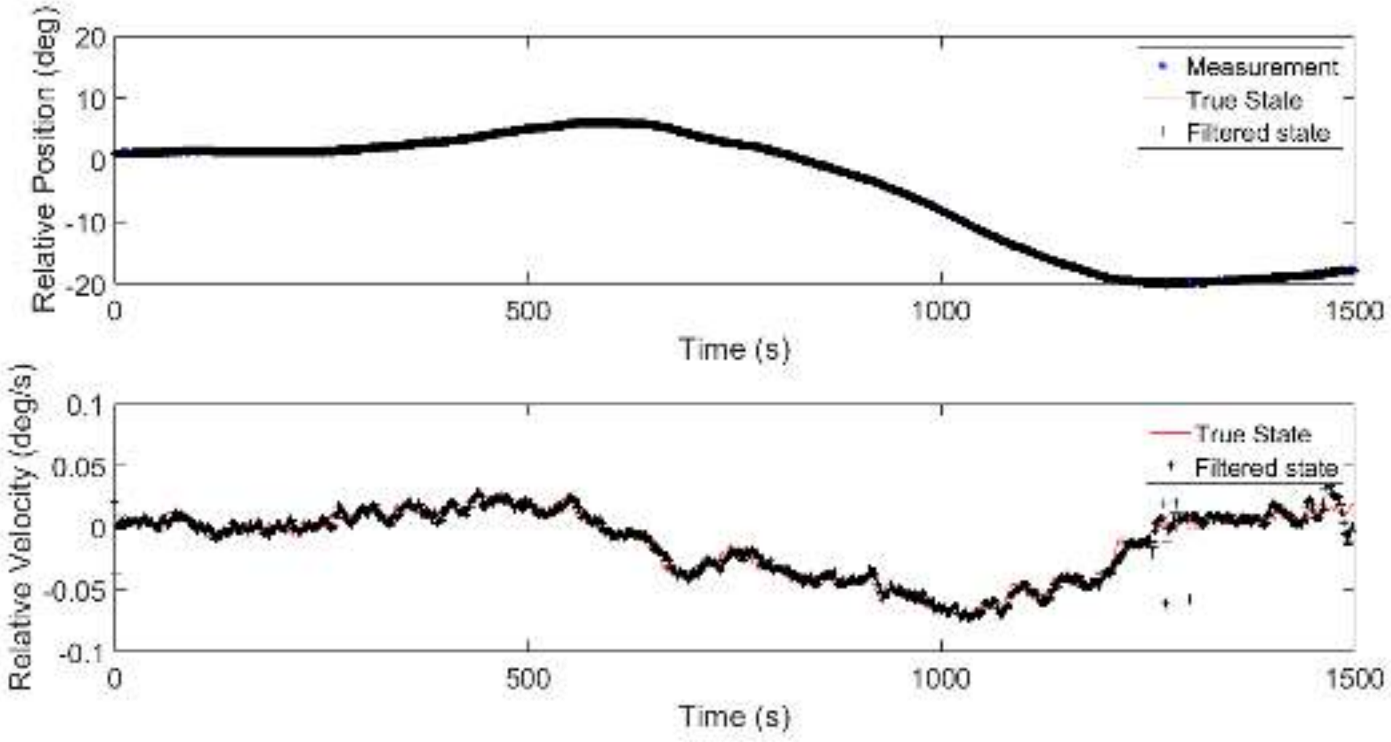
Case study II → Results



32,5 Posit implementation starts diverging around 1250 seconds

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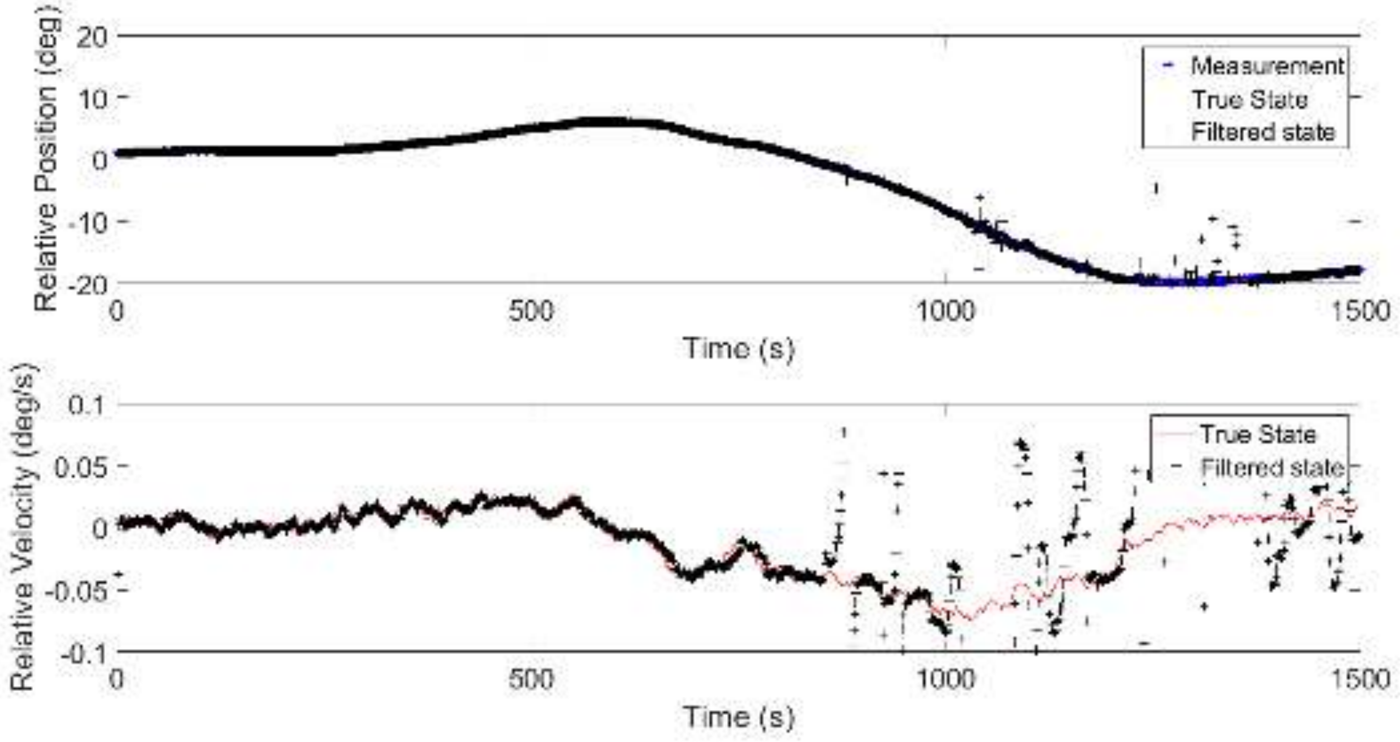
Case study II → Results



32,6 Posit implementation diverges even faster

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Empirical Analysis

$$\alpha(x) = log_{10}(\frac{x}{y - x})$$

x = any representable number on the number line

y = The next representable number after x

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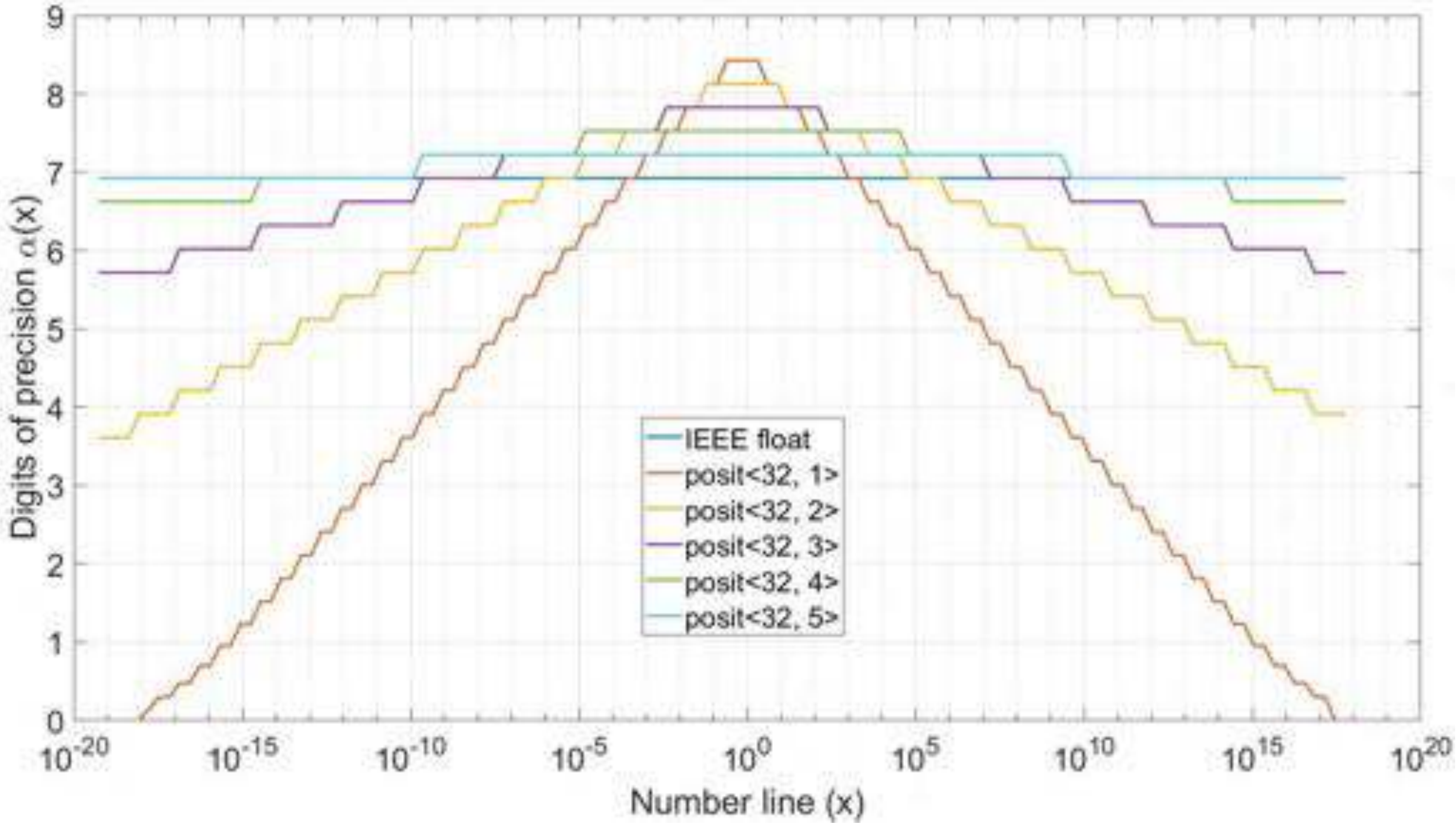
Precision of IEEE floats and Posit



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Empirical Analysis

$$\alpha(x) = log_{10}(\frac{x}{y - x})$$

x = any representable number on the number line

y = The next representable number after x

$$\mu = \frac{\int_{log(a)}^{log(b)} \alpha(x)dx}{\int_{log(a)}^{log(b)} dx}$$

a = Smallest number in application range

b = Largest number in application range

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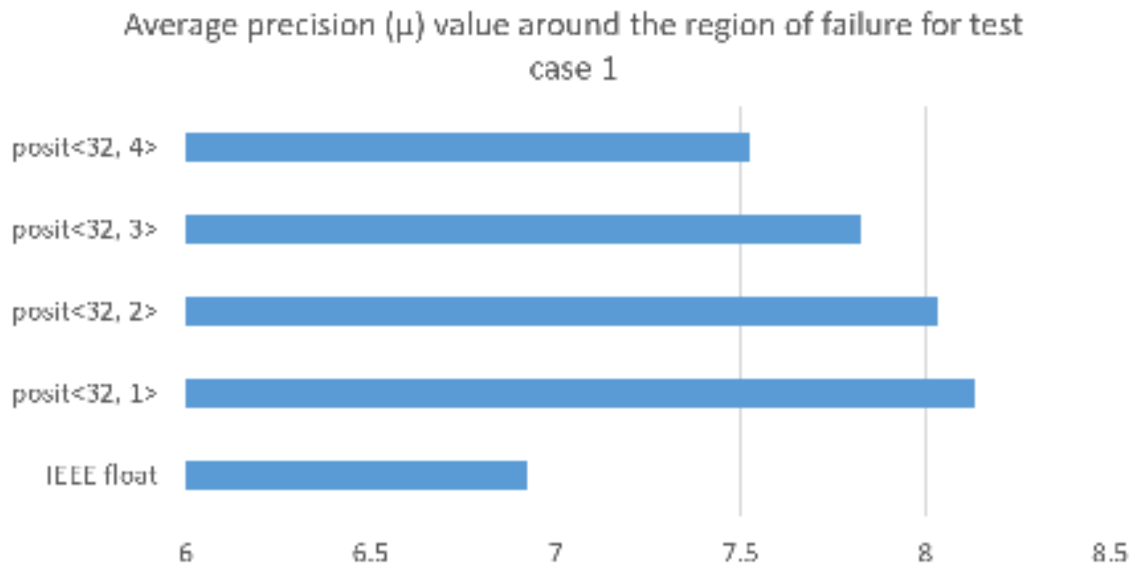
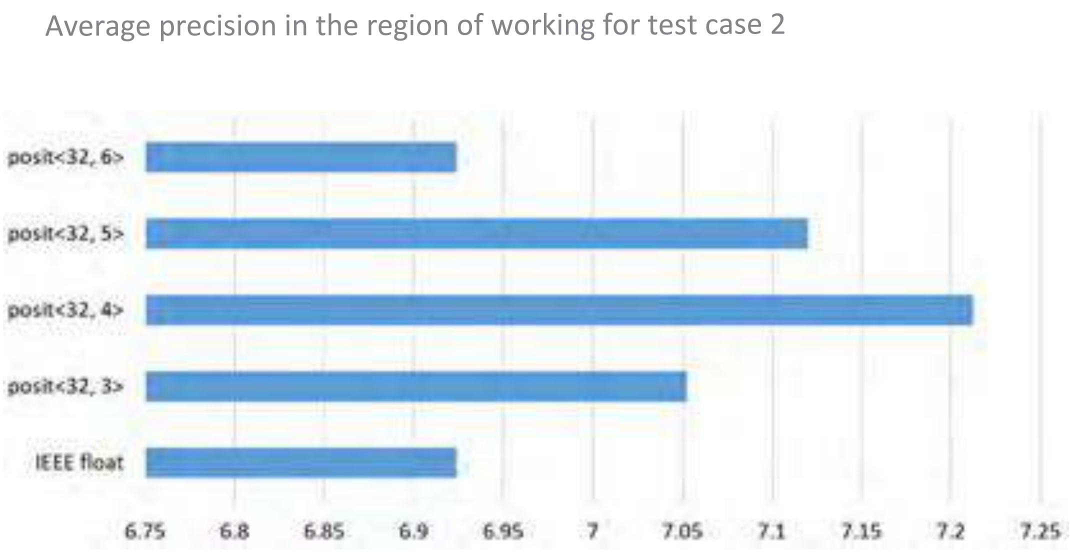
Empirical Analysis





Choosing configuration with highest average precision in measurement value region correlates to better stability of KF

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Summary

- Kalman filters are ubiquitously used to filter sensor noise but suffer from numerical instability
- Different mechanization try to solve the issue but are computationally expensive
- Similarly sized Posits can perform better than IEEE floats and maintain conventional KF stability
- Choice of Posit configuration is important
- If measurement data and simulation environment are present, the right configuration can be estimated by finding the average precision in the working/instable region of filter
- Benefits can potentially extend further with the use of Quires

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THANK YOU!

Questions?

You may reach me at vinayshankar.Saxena@in.bosch.com for any further queries